

Paper Reference 9FM0/4C
Pearson Edexcel
Level 3 GCE

Further Mathematics

Advanced

PAPER 4C: Further Mechanics 2

Monday 24 June 2024 – Afternoon

Time: 1 hour 30 minutes

**DO NOT RETURN THIS QUESTION
PAPER WITH THE ANSWER
BOOKLET AT THE END OF THE
EXAMINATION.**

Y75691A

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

YOU MUST HAVE

Mathematical Formulae and Statistical Tables (Green), calculator

YOU WILL BE GIVEN

Diagram Booklet

Answer Booklet

Turn over

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Booklet – there may be more space than you need.

Do NOT write on this Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Unless otherwise indicated, whenever a value of g is required, take

$g = 9.8 \text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Turn over

INFORMATION

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

There are 7 questions in this Question Paper.

The total mark for this paper is 75.

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

You may be provided with a model for Question 6(c).

It is NOT accurate.

Turn over

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

Turn over

1. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A particle P moves along a straight line.

Initially P is at rest at the point O on the line.

(continued on the next page)

Turn over

1. continued.

At time t seconds, where $t \geq 0$

- the displacement of **P** from **O** is
X metres
- the velocity of **P** is $v \text{ m s}^{-1}$ in the
positive **X** direction
- the acceleration of **P** is
 $\frac{96}{(3t + 5)^3} \text{ m s}^{-2}$ in the positive
X direction

(continued on the next page)

Turn over

1. continued.

(a) Show that, at time t seconds,

$$v = p - \frac{q}{(3t + 5)^2}, \text{ where}$$

p and q are constants to be determined.

(4 marks)

(b) Find the limiting value of v as t increases.

(1 mark)

(continued on the next page)

Turn over

1. continued.

(c) Find the value of x when

$$t = 2$$

(4 marks)

(Total for Question 1 is 9 marks)

Turn over

2. Refer to the Diagram for Question 2 in the Diagram Booklet.

A uniform rod of length $28a$ is cut into seven identical rods each of length $4a$

These rods are joined together to form the rigid framework **ABCDEA** shown in the diagram.

All seven rods lie in the same plane.

(continued on the next page)

2. continued.

The distance of the centre of mass of the framework from **ED is **d****

(a) Show that

$$d = \frac{8\sqrt{3}}{7}a$$

(4 marks)

The weight of the framework is **W**

The framework is freely pivoted about a horizontal axis through **C**

(continued on the next page)

Turn over

2. continued.

The framework is held in equilibrium in a vertical plane, with AC vertical and A below C , by a horizontal force that is applied to the framework at A

The force acts in the same vertical plane as the framework and has magnitude F

**(b) Find F in terms of W
(3 marks)**

(Total for Question 2 is 7 marks)

Turn over

3. Refer to Diagram 1 and Diagram 2 for Question 3 in the Diagram Booklet.

They show a hemispherical bowl of internal radius $10d$ that is fixed with its circular rim horizontal.

Diagram 1 shows a 3D view.

Diagram 2 shows a 2D side view.

The centre of the circular rim is at the point O

A particle P moves with constant angular speed on the smooth inner surface of the bowl.

(continued on the next page)

Turn over

3. continued.

The particle P moves in a horizontal circle with radius $8d$ and centre C

- (a) Find, in terms of g , the exact magnitude of the acceleration of P**
(6 marks)

The time for P to complete one revolution is T

- (b) Find T in terms of d and g**
(3 marks)

(Total for Question 3 is 9 marks)

Turn over

4. Refer to the diagram for Question 4 in the Diagram Booklet.

A uniform lamina **OAB** is in the shape of the region **R**

Region **R** lies in the first quadrant and is bounded by the curve with equation

$$\frac{x^2}{16} + \frac{y^2}{36} = 1, \text{ the } x\text{-axis, and the}$$

y -axis, as shown shaded in the diagram.

(continued on the next page)

4. continued.

The point A is the point of intersection of the curve and the x -axis.

The point B is the point of intersection of the curve and the y -axis.

One unit on each axis represents 1 metre.

The area of R is 6π

(continued on the next page)

Turn over

4. continued.

The centre of mass of R lies at the point with coordinates $(\bar{x}, \bar{y})$

(a) Use algebraic integration to show that

$$\bar{x} = \frac{16}{3\pi}$$

(5 marks)

(b) Use algebraic integration to find the exact value of $\bar{y}$
(4 marks)

(continued on the next page)

Turn over

4. continued.

The lamina is freely suspended from **A and hangs in equilibrium with **OA** at angle θ° to the downward vertical.**

(c) Find the value of θ
(3 marks)

(Total for Question 4 is 12 marks)

Turn over

5. A particle **P** moves in a straight line with simple harmonic motion about a fixed point **O**

The magnitude of the greatest acceleration of **P** is 18 m s^{-2}

When **P** is 0.3 metres from **O**, the speed of **P** is 2.4 m s^{-1}

(continued on the next page)

5. continued.

**The amplitude of the motion is
 a metres.**

(a) Show that

$$a = 0.5$$

(5 marks)

(b) Find the greatest speed of P

(2 marks)

(continued on the next page)

Turn over

5. continued.

During one oscillation, the speed of P is at least 2 m s^{-1} for S seconds.

(c) Find the value of S
(4 marks)

(Total for Question 5 is 11 marks)

Turn over

- 6. In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Refer to the diagram for Question 6 in the Diagram Booklet.

The shaded region, shown in the diagram, is bounded by the x -axis, the line with equation $x = 6$, the line with equation $y = 2$ and the y -axis.

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Turn over

6. continued.

This region is rotated through 360° about the x -axis to form a solid of revolution.

This solid is used to model a non-uniform cylinder of height 6 cm and radius 2 cm

The mass per unit volume of the cylinder at the point (x, y, z) is $\lambda(x + 2) \text{ kg cm}^{-3}$, where $0 \leq x \leq 6$ and λ is a constant.

(continued on the next page)

Turn over

6. continued.

(a) Show that the mass of the cylinder is

$$\mathbf{120\lambda\pi \text{ kg}}$$

(3 marks)

(b) Show that the centre of mass of the cylinder is

$$\mathbf{3.6 \text{ cm from O}}$$

(4 marks)

(continued on the next page)

Turn over

6. continued.

**Refer to the diagram for Question 6(c)
in the Diagram Booklet.**

**You may be provided with a
3D model.**

It is NOT accurate.

The point **O is the centre of one end
of the cylinder.**

The point **A is the centre of the other
end of the cylinder.**

(continued on the next page)

Turn over

6. continued.

A uniform solid hemisphere of radius 3 cm has density $\lambda \text{ kg cm}^{-3}$
The hemisphere is attached to the cylinder with the centre of its circular face in contact with the point A on the cylinder to form the model shown in the diagram for Question 6(c).

(continued on the next page)

Turn over

6. continued.

The model is placed with the end containing O on a rough inclined plane which is inclined at angle α° to the horizontal.

The plane is sufficiently rough to prevent the model from sliding.

The model is on the point of toppling.

**(c) Find the value of α
(6 marks)**

(Total for Question 6 is 13 marks)

Turn over

- 7. Refer to the diagram for Question 7 in the Diagram Booklet.**

A smooth solid hemisphere has radius r and the centre of its plane face is O

The hemisphere is fixed with its plane face in contact with horizontal ground, as shown in the diagram.

A small stone is at the point A , the highest point on the surface of the hemisphere.

The stone is projected horizontally from A with speed U

(continued on the next page)

Turn over

7. continued.

The stone is still in contact with the hemisphere at the point B , where OB makes an angle θ with the upward vertical.

The speed of the stone at the instant it reaches B is v

The stone is modelled as a particle P and air resistance is modelled as being negligible.

- (a) Use the model to find v^2 in terms of U , r , g and θ**
(3 marks)

(continued on the next page)

Turn over

7. continued.

When **P** leaves the surface of the hemisphere, the speed of **P** is **W**

Given that

$$U = \sqrt{\frac{2rg}{3}}$$

(b) show that

$$W^2 = \frac{8}{9}rg$$

(5 marks)

(continued on the next page)

Turn over

7. continued.

After leaving the surface of the hemisphere, P moves freely under gravity until it hits the ground.

- (c) Find the speed of P as it hits the ground, giving your answer in terms of r and g**
(3 marks)

(continued on the next page)

Turn over

7. continued.

At the instant when P hits the ground it is travelling at α° to the horizontal.

**(d) Find the value of α
(3 marks)**

(Total for Question 7 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER
